# Tailored orthonormal bases for wavelet fusion

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#### RESUMEN

Este trabajo propone un criterio objetivo para el diseño de filtros FIR (Finite-duration Impulse Response), de manera que desde sus coeficientes se obtengan directamente las bases wavelet ortonormales (F) que son aplicadas para la fusión de imágenes de satélite. La determinación del orden del polinomio que compone el filtro FIR, esta basada en el cálculo de la energía de la imagen a ser fusionada (imagen fuente) y la frecuencia de corte es obtenida de la determinación de un valor óptimo de entropía de estas imágenes. En este sentido, se ha determinado una base wavelet ortonormal particular para cada imagen fuente. Por otro lado, este criterio ha permitido la integración balanceada de la información contenida en las imágenes fuente en la imagen fusionada.

El algoritmo propuesto, ha sido evaluado con imágenes registradas por el sensor ETM+ (multiespectral, MULTI y pancromático, PAN) del Landsat 7. Los resultados obtenidos muestran que las bases wavelet ortonormales particularizadas para cada imagen fuente presentan una mejor relación señal-ruido (SNR), lo que redunda en una mejor integración, tanto espacial como espectral, de la información proveniente de las imágenes fuente en comparación con las bases wavelet estándar.

PALABRAS CLAVE: filtro FIR, entropía de la información, fusión de imágenes, energía de la imagen, bases wavelet.

#### ABSTRACT

This work proposes an objective criterion for FIR (Finite-duration Impulse Response) digital filters design, whose coefficients have been used to directly built the orthonormal wavelet bases (F) to be applied to satellite imagery fusion. The determination of the filter order was based on the calculation of the images power to be fused (source images) and the cut-off frequency has been obtained from the optimum entropy level of these images. In this way, a tailored orthonormal basis has been determined for each source images. Besides, this criterion has allowed to integrate in a balanced way the amount of information provided by each of the images to be fused.

The proposed algorithm has been tested with images registered by the Landsat 7 ETM+ (multispectral, MULTI, and panchromatic, PAN, images). The results obtained have shown that tailored orthonormal bases provides a better signal-to-noise ratio (SNR), what gives rise to a better integration of the spatial and spectral information from the sources images than standard orthonormal bases.

KEY WORDS: FIR filters, information entropy, image fusion, power image, wavelet bases.

## **INTRODUCCIÓN**

Remote Sensing has many geoscience applications which require satellite imagery displaying spatial and spectral high resolution simultaneously, in order to define thematic and spectral classes able to properly characterising the surfaces being studied. However, the massive use satellite sensors (Landsat TM, AVHHR, ASTER, etc.) only provide one of these features, excluding the other. Although nowadays there exist some satellites like Quickbird or IKONOS that produce high spatial and spectral resolution images, the costs to obtain these images are very high. One way of obtaining images of high spatial and spectral resolution, at affordable costs, is through the image fusion techniques.

Fusion of multiple sensors data presents some advantages over obtaining data from an only sensor. Given that the information has been optimally fused, one of the most important things is to have more than one observer of the phenomenon being analysed. In this way, more and better information can be integrated for further processing. There are several works that describe different techniques and definitions for image fusion (Wald, 1999; Pohl, 1999; Shettigara, 1992; Bretschneider, 2000).

One of the most widely used methods is the image fusion through a hierarchical decomposition by means of wavelet transform, such as the Mallat's algorithm. An image is decomposed into a set of multiresolution images and their associated wavelet coefficients. The coefficients, corresponding to each level, contain the spatial differences (detail) between two successive resolution levels (Ranchin and Wald, 2000; Din-Chang et al., 2001).

The essential idea of the fusion based on the wavelet transform is to incorporate the high spatial frequency information of the panchromatic image (PAN), contained in its wavelet coefficients, into the degraded levels of the multispectral image (MULTI). This incorporation can be carried out by means of a direct replacement, addition or selection of the corresponding wavelet coefficients (Núñez et al., 1999) as it is displayed Fig. 1. In particular,  $d_2^{HL}$ , and  $d_2^{HL}$ , represents the horizontal, diagonals and verticals details of the image source, respectively, and they corresponds to the wavelet coefficients.



Figura 1. Standard scheme for wavelet image fusion.

Wavelet fusion is a computationally intensive process. This method allows that the colour distortion can be reduced to a certain extent, appearing the fused image like a result of a high pass filtering fusion, e.g., the colour seems not being smoothly integrated into the spatial features (Zhang, 2002).

One of the reasons of the problems mentioned in the previous paragraph, is the difficulty of establishing an objective methodology to determine tailored orthonormal bases (Coifman and Wickerhauser, 1992) for each spectral band i of the MULTI image,  $\Phi_{MULTI}$ , and for the PAN image,  $\Phi_{PAN}$ . Therefore, the main goal of this work is to propose an objective criterion that provides this kind of tailored orthonormal bases, for the fusion of a MULTI image and a PAN image. In the next section, a discussion about the determination of these bases for wavelet fusion is presented.

#### AN OBJECTIVE CRITERION FOR DETERMINING TAILORED WAVELET ORTHONORMAL BASES

Fusion strategies based on hierarchical wavelet decomposition can be classified in two groups (Núñez et al., 1999): (1) Those that determine wavelet coefficients by selection rules, applied to each spectral band of the MULTI image and the PAN image. (2) And those that partially replace the wavelet coefficients of the high resolution image by the corresponding ones of the low resolution image. These two strategies allow obtaining the wavelet coefficients for the fused image, considering predefined orthonormal bases. Nevertheless, it is possible to expect that tailored orthonormal bases, provide better quality results, since in this case, the wavelet coefficients will only depend on the features of the images to be fused.

It should be also noted that the fusion of images through wavelet transform can be considered as the successive application of low pass (LPF) and high pass (HPF) filters for the separation of the high and the low frequencies of the image. Based on the previously stated aspects, and as it has been already mentioned, a new strategy for the images fusion by means of the wavelet transform is proposed, based on the definition of bases  $\Phi_{_{MULTI}}$  and  $\Phi_{_{PAN}}.$  These bases are built from the coefficients of the designed FIR filters (LPF and HPF). A critical aspect that guarantees good spatial and spectral quality of the fused images through this technique is the appropriate tune-up of the LPF and HPF filters. This tune-up implies determining the order of filters (M)and the cut-off frequency  $(\omega_c)$  (Proakis and Monolakis, 1995). For this aim, this work proposes an objective criterion exclusively depending on the images to be fused. The order of the filters, M, has been determined through a power sensitivity analysis of the source images, with respect to the variation of  $\omega_{c}$ . And the optimum level of entropy of the source images has been used to determine the cutoff frequency,  $\omega_c$ , for the previously determined value of M. This criterion has allowed a balanced integration of the amount of information provided by each of the source images. This study must be performed for all spectral bands of a particular MULTI image and the PAN image.

Fig. 2(a) and (b) show the curves given by the M power sensitivity analysis for the spectral band ETM1 of the scene used in this work. It is obvious that for large values of M, the filtering results are almost independent of the cut-off frequency, preventing the wished control of the amount of filtered information. Since a value of M=4 provides a good sensitivity as opposed to the rest of the other analysed values, this value has been selected in this case. That means that the tailored wavelet bases are built by only five coefficients. And it implies a noteworthy reduction of computational requirements. The same analysis must be performed to determine M for the rest of the bands.

Once the order (*M*) has been specified, the common cut-off frequency of the lowpass and highpass FIR filters must be determined for each spectral band. Since these values will determine the amount of information that every source image will bring to the fused image, the entropy of the source images has been calculated for different values of  $\omega_c$ , according to equation (1) (Price, 1987):

$$H = -\sum_{j=0}^{DN_{max}} p_j \log_2 p_j \tag{1}$$

Where  $p_j$  represents the probability of the values of the digital numbers (*DN*) present in the image.

Fig. 3 shows the variation of entropy in the MULTI (ETM1, ETM2, ETM3, ETM4, ETM5 and ETM7) and PAN images for different  $\omega c$  values and for *M*=4.

The intersection points between the curves corresponding to each band of the MULTI image and the curve of the PAN image (Fig. 3), give the values of  $w_c$ , for the different spectral bands. That assures that the fused images contain the same amount of information coming from each one of the source images, and also it indicates that depending on considered spectral band, the weight of the PAN image into the original MULTI image is different. Now, the required information (M and  $\omega_c$ ) to calculated the FIR filters coefficients,  $B_i$  (equation. 2), are available (Proakis and Manolakis, 1995).

$$y(n) = B_0 x(n) + B_1 x(n-1) + \dots + B_M x(n-M)$$
(2)

Where x(i), with i varying from n to *n*-*M*, corresponds to the discrete components of the input signal, y(n) corresponds to the output signal.

Since, it is well known that the Bi coefficients determine orthonormal bases (Proakis and Monolakis, 1995), these values will be employed to built directly the tailored orthonormal bases to be used in this wavelet image fusion method.



Figura 2. FIR filter order determination for a particular spectral band: (a) Low pass, (b) High pass.



Figura 3. FIR filters cut-off frequency ( $\omega_c$ ) per band.

#### RESULTS

The data used for the evaluation of the proposed method were two 4.5km x 4.5km scenes located in Madrid, Spain. The images MULTI and PAN were collected by the sensor ETM+ (Landsat 7) on 20th August, 1999. The ETM1, ETM2, ETM3, ETM4, ETM5 and ETM7 bands of the Thematic Mapper ETM+ sensor, with a common 30m spatial resolution, were fused with the PAN (ETM8) image, with a 15m spatial resolution, obtaining a new multispectral image distinguished by a high spatial (15m) and spectral (6 bands) quality.



Figura 4. Overviwe of the proposed fusion scheme for a particular band.

The methodology described in section 2 must be applied to each band of the multispectral image. The images to be fused must be radiometrically corrected and spatially referenced. In order to obtain better results, it is advisable to reference the low spatial resolution image in relation to the high spatial resolution image (Price, 1999).

Fig. 4 shows an overview of the fusion methodology proposed in this work, applied to the ETM1 and PAN images of the data described above. The first step in the fusion process is to resize the multispectral image to the PAN image size, by means of a interpolation methods. In the next step, the PAN and MULTI images should be wavelet transformed through the tailored orthogonal bases previously determined, in order to obtain one dyadic level wavelet decomposition. Finally, MULTI high-frequency components are replaced by PAN high-frequency components, then the inverse wavelet transform is applied to obtain the  $i^{th}$  band of the fused image.

	ETM1	ETM2	ETM3	ETM4	ETM5	ETM7
М	4	4	4	4	4	4
ω	0.30	0.25	0.18	0.22	0.14	0.16

Tabla 1. M and  $\omega_c$  value, for FIR filters design

		Coeff. 1	Coeff. 2	Coeff. 3	Coeff. 4	Coeff. 5
BAND 1	$\Phi_{_{MULTI_1}}$ $\Phi_{_{PAN_1}}$	0.0121 -0.0121	0.1386 -0.1386	0.2988 0.7012	0.1386 -0.1386	0.0121 -0.0121
BAND 2	$\Phi_{_{MULTI_2}} \\ \Phi_{_{PAN_2}}$	0.0127 -0.0127	0.1215 -0.1215	0.2500 0.7500	0.1215 -0.1215	0.0127 -0.0127
BAND 3	$\Phi_{_{MULTI_3}}$ $\Phi_{_{PAN_3}}$	0.0115 -0.0115	0.0919 -0.0919	0.1797 0.8203	0.0919 -0.0919	0.0115 -0.0115
BAND 4	$\Phi_{_{MULTI_4}} \\ \Phi_{_{PAN_4}}$	0.0125 -0.0125	0.1090 -0.1090	0.2187 0.7813	0.1090 -0.1090	0.0125 -0.0125
BAND 5	$\Phi_{_{MULTI_5}} \\ \Phi_{_{PAN_5}}$	0.0097 -0.0097	0.0725 -0.0725	0.1387 0.8613	0.0725 -0.0725	0.0097 -0.0097
BAND 7	$\Phi_{_{MULTI_7}}$ $\Phi_{_{PAN_7}}$	0.0107 -0.0107	0.0828	0.1602 0.8398	0.0828 -0.0828	0.0107 -0.0107

Tabla 2. Coefficients of  $\Phi_{_{\text{MULTI}_i}}$  and  $\Phi_{_{\text{PAN}_i}}$ 

Table 1 and Table 2 summarize the values of M and  $\omega_c$  (expressed as normalised frequency), as well as, the coefficients of the tailored orthonormal bases  $\Phi_{\text{MULTI}_i}$  and  $\Phi_{\text{PAN}_i}$ , respectively, obtained for the considered source images. Fig. 5(a), 5(b) and 5(c) show three RGB compositions (R=ETM7, G=ETM4 and B=ETM1). A scene of the original MULTI image (5a) and its corresponding scenes fused through the standard wavelet fusion method (5b) and the proposed method (5c). A visual analysis shows that both fused images presents a clear spatial resolution enhancement as compared to the original image. Besides, it can be appreciated smoother transitions in the high gradient areas in the image fused with the tailored wavelet bases than in the image fused with standard wavelet bases (Biorthogonal). That means that the tailored bases provide a better integration of the spatial and spectral information form the source images.

Different quality indices, proposed in the literature, have been calculated with the aim of quantifying the quality of the images fused by the proposed method: the signal-to-noise ratio (SNR), correlation and ERGAS (Wald et al., 1997). Besides, those indices have allowed to compared the fusion performances of the proposed method versus the standard wavelet fusion method, described in previous section. The SNR has been calculated (Table 3) for all bands of the MULTI, by means of the equations (3).

$$SNR^{*} = \frac{\sum_{x} \sum_{y} f(x, y)^{2}}{\sum_{x} \sum_{y} \left\{ f(x, y) - \hat{f}(x, y) \right\}}$$
$$SNR(dB) = 10 \cdot \log(SNR^{*})$$
(3)

Where f(x,y) and  $\hat{f}(x,y)$  are the *i*<sup>th</sup> band of the original MULTI and the fused image, respectively.



\*Figura 5. RGB compositions (R=ETM7, G=ETM4, B=ETM1) (a) Scene of the original MULTI image, (b) fused scene through the standard orthonormal basis and (c) fused scene through the tailored orthonormal bases.

SNR(dB)	ETM1	ETM2	ETM3	ETM4	ETM5	ETM7	Average
STANDARD BASIS	28.38	18.41	21.64	24.36	5.22	20.11	19.69
TAILORED BASES	29.92	17.47	22.14	24.90	8.99	20.58	20.67

Tabla 3. Signal-to-Noise Ratio (SNR)dB)).

It can be noted that, except for band ETM2, the proposed method provides a better signal-to-noise rate than the standard method, that confirms the aspects discussed in the visual analysis.

Table 4 shows the statistical correlation values between the spectral bands of the original image and the corresponding spectral bands of the fused image, obtained through tailored and standard bases. The statistical correlation is defined by equation (4):

$$corr\left(A_{B}^{\prime}\right) = \frac{\sum_{j=1}^{Npix} (A_{j} - A_{m})(B_{j} - B_{m})}{\sqrt{\sum_{j=1}^{Npix} (A_{j} - A_{m})^{2} \sum_{j=1}^{Npix} (B_{j} - B_{m})^{2}}}$$
(4)

Where  $A_m$  and  $B_m$  are the averages of the digital numbers of the corresponding images and  $N_{pix}$  is the number of pixels in the image.

It can be observed at Table 4 that correlation values of the fused images by both method are very close to 1 for all spectral bands, meaning that both methods provides similar good performances.

CORR	ETM1	ETM2	ETM3	ETM4	ETM5	ETM7
STANDARD BASIS	0.971	0.979	0.985	0.991	0.989	0.987
TAILORED BASES	0.979	0.983	0.985	0.991	0.989	0.987

Tabla 4. Correlation between original and fused images.

Finally, the ERGAS index has been used to quantify the spectral quality of fused images. It has been established that images with an ERGAS value lower than 3 exhibit a good spectral quality (Wald et al. 1997). Table 5 summarizes the index values calculated and averaged for the six bands of the

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fused image. The low ERGAS value given by the proposed method, evidence a light improvement in the spectral quality respect to the standard wavelet fusion method.

BASES	STANDARD	TAILORED
ERGAS	2.93	2.92

Tabla 5. ERGAS index.

## CONCLUSION

A new methodology, to determine tailored orthonormal wavelet bases for multispectral images fusion, has been proposed in the present work. These bases depend only upon features of the source images, freeing the process from other subjective factors. The methodology is based on an objective criterion for determining the parameters  $(M, \omega_c)$ implied in the tune up of the FIR filters.

In this study, it has been proved that low values of M provides a high power sensitivity of the image to be fused with respect to the variation of  $\omega_c$ . This high sensitivity has allowed to control the amount of information that each source image contributes to the fused image. On the other hand, the low values of M imply that a small number of coefficients is required to define the tailored orthonormal bases. This diminishes the computational requirements. Moreover the calculation of a common entropy value for the source images has allowed to determine an unique cut-off frequency for the low and high pass filters, but different to each band, guaranteeing that the same amount of the information is integrated from the source images in each band.

In other words, the influence of the PAN image in each of the fused bands is different. From the evaluation of the quality of the fused images, it is reasonable to conclude that tailored orthonormal bases provides a better integration of spatial and spectral information than image fused by the standard wavelet method. Since it has been obtained an ERGAS value lower than 3, it can assure that the fused image by the proposed method preserves the spectral features of the MULTI image. Finally, the evaluation of the signal-to-noise ratio (SNR) has shown that tailored orthonormal bases provides a better integration of the spatial and spectral information from the sources images than standard orthonormal bases.

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# NOTICIAS

# Socios numerarios y correspondientes estudiantes

El artículo 15° de los estatutos de la Asociación Española de Teledetección (A.E.T.) contempla la posibilidad de que los socios numerarios (residentes en territorio español) y correspondientes (no residentes en territorio español), estudiantes de Facultades, Escuelas Técnicas y Universitarias, gocen de un 50% de bonificación en sus cuotas.

Animamos a todos los estudiantes, con interés en conocer las investigaciones y técnicas de teledetección, para que se integren como socios de la Asociación Española de Teledetección (A.E.T.) por una cuota anual de 18 euros.